SUPERVISORY ADAPTIVE BALANCING OF RIGID ROTORS DURING ACCELERATION

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ABSTRACT

This paper investigates active real-time balancing of rigid rotors during the transient time of acceleration. First, the governing equation of motion of a rigid rotor is formulated. A parametric adaptive control scheme is then designed based on this dynamic model. An ordinary recursive least squares estimation method is used to estimate parameters in this adaptive control scheme. A supervisory control strategy is proposed to coordinate the parameter estimation and the balancer control action. Simulation results show that the whole scheme can efficiently reduce the imbalance-induced vibration in the rigid rotor-bearing system.

NOMENCLATURE

m	mass of the shaft
k	spring rate of the bearings
c	damping coefficient of the bearings
m_u	imbalance mass of the rotor
m_b	imbalance mass provided by the balancer
u_x, u_y, u_z	coordinates (in body-fixed coordinate) of the imbalance
w_x, w_y, w_z	coordinates (in body-fixed coordinate) of the imbalance provided by the balancer
L, R	length and radius of the shaft
I_p, I_t	polar moment of inertia and diametric moment of inertia of the shaft
R_X, R_Z	displacements (in inertial coordinate) of the mass center of the shaft in X and Z directions
$\dot{R}_X, \dot{R}_Z$	velocities of the mass center of the shaft
$\ddot{R}_X, \ddot{R}_Z$	accelerations of the mass center
$\phi, \dot{\phi}, \ddot{\phi}$	rotating angle, rotating speed, and rotating angular acceleration of the shaft
ψ, θ	Euler angles to describe the orientation of the body-fixed coordinate frame in the inertial coordinate frame. (ψ, θ, ϕ) forms an Euler angle set.

1. INTRODUCTION

Rotating machinery, including machining spindles, industrial turbomachinery, and aircraft gas turbine engines, are very commonly used in industry. Vibration caused by mass imbalance is an important factor limiting the performance and fatigue life of the rotating system. Therefore, a balancing procedure is necessary for rotating systems. Off-line balancing methods (Wowk, 1995) are widely used in practice. However, off-line balancing methods cannot deal with the situation of imbalance distribution changing during operation. This could happen when the rotor is handling abrasive or sticky materials. Some researchers (Gosiewski, 1985; Gosiewski, 1987; Van De Vegte and Lake, 1978; Van De Vegte, 1981) try to actively balance the rotating systems during operation by using mass redistribution devices. Magnetic bearings are used by other researchers (Knospe, et al., 1996; Lum, et al., 1996) to suppress the imbalance-induced vibration or the force transmitted to the base. Their research concentrates on the constant rotating speed case: the so-called "steady state" case. The dynamic response of a rotor under this condition is simple. Influence coefficient method is used in the active balancing scheme.

In some other cases, the balancing needs to be completed during speed-varying transient time. A couple examples are given as follows.

- In high-speed machining, the machining tool will be engaged in cutting as soon as the spindle goes into steady state. The active balancing has to be done during the acceleration period to avoid increasing the cutting cycle time.

- Large turbomachinery usually works at a speed above its first or second critical speeds. To avoid the large vibration that occurs when it passes through its critical speeds, balancing during acceleration is needed.

Unfortunately, this problem is not well studied in the literature. In this paper, an active balancing scheme that can balance the rotor-bearing system during acceleration transient time will be developed based on a rigid rotor model.

The actuator used in this research is an innovative mass redistribution device. This device is mounted on the rotor. It can be viewed as a concentrated mass that can rotate with the rotor, as well as be controlled to change its position on the rotor. The power and control commands are transferred from the stationary part by a rotary transformer inductively across an air gap. The details of this device can be found in Dyer, et al. (1998).

This paper consists of five sections. In section 2, a speed-varying transient dynamic model of a rigid rotor will be given. Instead of being modeled as static and dynamic imbalance (Genta, 1993), the imbalance of the rotor is modeled as a concentrated mass. This model is more suitable for estimation setup and control design for the specified actuator used in this research. A recursive least squares estimation procedure will be presented in section 3. The imbalance in the rotating system will be estimated using the transient vibration signal. Section 4 will give a supervisory adaptive control strategy that is efficient for reducing the imbalance-induced vibration. Finally, conclusions are presented in the last section.

2. SPEED VARYING DYNAMIC MODEL OF A ROTOR-BEARING SYSTEM

Many rotors can be modeled as rigid. The geometric setup of a rigid rotor model is shown in Figure 1.

The basic assumptions in this rigid rotor model are listed as follows.

A1. The rotor is a rigid shaft with a circular cross section and the imbalance is a concentrated mass on the shaft.

A2. The bearings are isotropic and modeled as a set of linear springs and dampers. The bearings are located at each end of the shaft. The mass center of the rotor is located at the midpoint between the two bearings. This simplifies the dynamic model, but the extension to a more general rigid rotor model is straightforward.

A3. The angular acceleration of the axial rotational motion is assumed to be a known constant. Its value can be obtained from the speed profile of the rotor.

A4. The lateral vibration motions are assumed to be small to simplify the dynamics. The translational motion in Y direction is assumed to be zero.

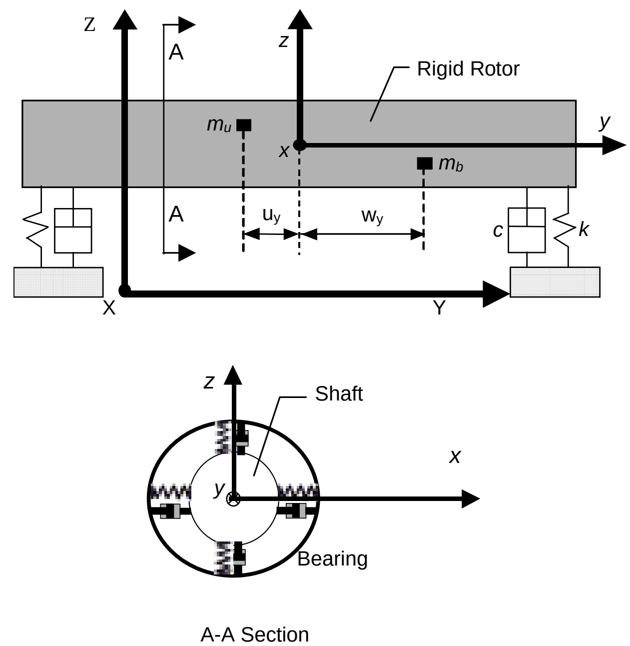

FIGURE 1. GEOMETRIC SETUP OF THE RIGID ROTOR.

Two coordinate systems are used in the derivation: the body-fixed coordinate *xyz* and the inertial coordinate XYZ. The body-fixed *y*-axis is picked as the rotating axis of the shaft and *x*- and *z*-axes are defined by the other two principal inertia axes of the rotor. The origin of *xyz* is selected as the geometric center of the shaft. The XYZ coordinate system is the stationary inertial frame and coincides with the *xyz* coordinate system when the rotor is at rest. The position of the origin of the body-fixed coordinate, i.e., the geometric center of the rigid rotor, is defined by (R_x, R_y, R_z). Usually, there is structural constraint on the translational motion in the axial direction. Therefore, R_y is assumed to be zero. The orientation of the body-fixed coordinate is represented by three Euler angles (ψ, θ, ϕ).

The governing equation of motion (Eq. (1)) can be obtained by Newton's law and Euler's equations of motion for rigid body. It has been simplified by small motion assumptions. A brief derivation can be found in the Appendix.

$$\begin{cases} \ddot{R}_x = -\frac{2k}{m}R_x - \frac{2c}{m}\dot{R}_x + \frac{-m_b w_z - m_u u_z}{m}f_1 + \frac{m_b w_x + m_u u_x}{m}f_2 \\ \ddot{R}_z = -\frac{2k}{m}R_z - \frac{2c}{m}\dot{R}_z + \frac{m_b w_x + m_u u_x}{m}f_1 + \frac{m_b w_z + m_u u_z}{m}f_2 \\ \ddot{\theta} = -\frac{kL^2}{2I_t}\theta + \frac{I_p\ddot{\phi}}{I_t}\psi - \frac{cL^2}{2I_t}\dot{\theta} + \frac{I_p\dot{\phi}}{I_t}\dot{\psi} + \frac{m_b w_x w_y + m_u u_x u_y}{I_t}f_1 + \frac{m_b w_y w_z + m_u u_y u_z}{I_t}f_2 \\ \ddot{\psi} = -\frac{I_p\ddot{\phi}}{I_t}\theta - \frac{kL^2}{2I_t}\psi - \frac{I_p\dot{\phi}}{I_t}\dot{\theta} - \frac{cL^2}{2I_t}\dot{\psi} + \frac{m_b w_y w_z + m_u u_y u_z}{I_t}f_1 + \frac{-m_b w_x w_y - m_u u_x u_y}{I_t}f_2 \end{cases} \quad (1)$$

where $f_1 = \ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi, f_2 = \ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi$, are known functions of time. The model is a linear time variant model because the speed $\dot{\phi}$ is varying.

There are two natural vibration modes of the rigid rotor: the deflection motion and the inclination motion. The deflection motion is the displacement of the mass center of the rotor in the OXZ plane. The inclination motion is the change of the orientation of rotor with respect to the OXYZ coordinate.

In the next section, a recursive least squares method based on this model will be used to estimate the system imbalance using the speed varying transient vibration signals.

3. ESTIMATION OF IMBALANCE USING TRANSIENT VIBRATION SIGNAL

The objective of active balancing is to reduce the imbalance-induced vibration. The position and magnitude of the imbalance in the rotor system must first be estimated. The balancer can then provide imbalance to counteract the system imbalance. In this paper, a least squares estimation method is used to estimate the imbalance based on the transient vibration signal.

The general model of the least squares estimation method is:

$$y = \mathbf{X}^{\mathbf{T}}\boldsymbol{\theta} + \boldsymbol{\varepsilon}, \quad (2)$$

where the observations are y**,** the explanatory variables are **X**, the unknown parameters are $\boldsymbol{\theta}$, and the noise following a normal distribution is $\boldsymbol{\varepsilon}$. It is well known that the least squares solution to the above equation is

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}\mathbf{X}^T)^{-1}\mathbf{X}y\,. \quad (3)$$

This is the formula for batch-mode estimation. By performing the matrix operations, the equation can be solved recursively (Ljung and Soderstrom, 1983).

The equations used by the least squares estimation are selected from Eq. (1). Among all the parameters in Eq. (1), the rotational motion of the shaft ($\phi, \dot{\phi}, \ddot{\phi}$), the geometric parameters (L, R, I_p, I_t), and the mass of the rotor (m) are assumed known. This information can be obtained from rotating speed measurements and the geometric configuration of the rotor. Based on the rigid rotor assumption, the states of the rotor $(R_x, R_z, \theta, \psi, \dot{R}_x, \dot{R}_z, \dot{\theta}, \dot{\psi})$ can be obtained by measuring the motions of two points that are not in the same transverse plane. Hence, we assume these variables also known. The unknown parameters to be estimated are the dynamic parameters of the system (*c, k*) and the imbalance (m_u, u_x, u_y, u_z).

To set up the identification problem, the continuous model needs to be transformed into a discrete model. One way to get an approximated discrete system at *t=kT* (*T* is the sampling interval) is to approximate the derivative of a variable **r**(t) as

$$\dot{\mathbf{r}}(t)\big|_{t=kT} \approx \frac{1}{T}[\mathbf{r}((k+1)T) - \mathbf{r}(kT)]\,. \quad (4)$$

The smaller the sampling interval, the more accurate this transformation will be.

Since the geometry of the rotor is symmetric and the bearings are isotropic, the vibrations in X and Z directions provide the same amount of information. Hence, only the first and the third equations of Eq. (1) are used in the parameter estimation scheme.

If let $a_1 = \frac{k}{m}, a_2 = \frac{c}{m}, b_1 = \frac{m_u u_x}{m},\ b_2 = \frac{m_u u_z}{m}$, $b_3 = \frac{m_u u_x u_y}{m}, b_4 = \frac{m_u u_z u_y}{m}$ be unknown parameters and $b_0 = \frac{1}{m}, g_0 = \frac{m}{I_t} = \frac{12}{3r^2 + L^2}, g_1 = \frac{mL^2}{2I_t} = \frac{6L^2}{3r^2 + L^2}$, $g_2 = \frac{I_p}{I_t} = \frac{6r^2}{3r^2 + L^2}$ are known parameters, the first and the third equation of Eq. (1) are

$$\begin{aligned} \dot{R}_x[(k+1)T] &= T[(\frac{1}{T} - 2a_2)\dot{R}_x - 2a_1 R_x + \\ &\quad (m_b w_x f_2 - m_b w_z f_1)b_0 + b_1 f_2 - b_2 f_1]\big|_{t=kT} \quad (5) \\ \dot{\theta}[(k+1)T] &= T[(\frac{1}{T} - g_1 a_2)\dot{\theta} - g_1 a_1 \theta + g_2\ddot{\phi}\psi + g_2\dot{\phi}\dot{\psi} + \\ &\quad b_0 m_b w_y g_0(w_x f_1 + w_z f_2) + b_3 g_0 f_1 + b_4 g_0 f_2]\big|_{t=kT} \end{aligned}$$

By collecting known and unknown variables in Eq. (5), we can formulate a linear regression problem. Letting

$$y_1 = \frac{\dot{R}_X[(k+1)T] - \dot{R}_X[kT]}{T} - (m_b w_x f_2 - m_b w_z f_1) b_0 \big|_{t=kT}$$

$$\mathbf{x_1} = \begin{bmatrix} -2R_X & -2\dot{R}_X & f_2 & f_1 \end{bmatrix}^T \big|_{t=kT} ,$$

$$\varphi_1 = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix}^T$$

and

$$y_2 = \frac{\dot{\theta}[(k+1)T] - \dot{\theta}\big|_{t=kT}}{T}$$

$$- [g_2 \ddot{\phi}\psi + g_2 \dot{\phi}\dot{\psi} + b_0 m_b w_y g_0 (w_x f_1 + w_z f_2)] \big|_{t=kT} ,$$

$$\mathbf{x_2} = \begin{bmatrix} -g_1\theta & -g_1\dot{\theta} & g_0 f_1 & g_0 f_2 \end{bmatrix}^T \big|_{t=kT}$$

$$\varphi_2 = \begin{bmatrix} a_1 & a_2 & b_3 & b_4 \end{bmatrix}^T$$

we can obtain

$$y_1 = \mathbf{x_1}^T \varphi_1 + \varepsilon_1 , \qquad y_2 = \mathbf{x_2}^T \varphi_2 + \varepsilon_2 \tag{6}$$

where ε_1 and ε_2 are normally distributed noise which include multiple noise sources, such as the measurement noise and modeling noise. Based on these two equations, a_1, a_2, and b_1~b_4 can be estimated by using the recursive least squares estimation algorithm.

A simulation study is conducted on this recursive least squares method. The parameters used in the simulation are listed in table 1.

TABLE 1. PARAMETERS USED IN SIMULATION

L	R	c	K	m_u	u_x	u_y	u_z
0.5	0.1	10^3	10^7	0.5	0.08	0.05	0.05

All numbers in the table are in SI units. The balancer has imbalance magnitude 0.5kg and is located at 0.05m along the y-axis from the mass center. The angular acceleration is 500 rad/sec^2. At the starting point, the balancer is held at [0, 0.05, 0]. Only the centrifugal force caused by the imbalance of the balancer is considered. Hence, the balancer movement can be modeled as a sudden jump of the imbalance without causing the tangential force. In the simulation, at time *t=0.3* seconds, the imbalance provided by the balancer (0.5kg) jumps to [0.08 0.05 –0.05] and stays at that position to the end of the simulation.

The outputs, [X_a, Z_a, X_b, Z_b], are the displacements of two points on the rotating axis. The distances of these two points from the mass center are *0.1m* and *–0.08m*, respectively. A measurement noise with variance=10^{-6}m^2 is added onto the outputs. Eq. (6) is used to setup the estimation scheme. The displacement variables in Eq. (6) can be calculated from the outputs under the rigid rotor assumption. Eq. (4) is used to obtain the velocity and acceleration variables. The input of the system is taken as the position of the balancer.

Figure 2 shows the estimation results of b_1 ~ b_4. One significant characteristic of these results is that after the movement of the balancer, which excites the system, the estimation values have a jump toward the true values. To obtain an even faster convergence rate, more balancer movements can be used.

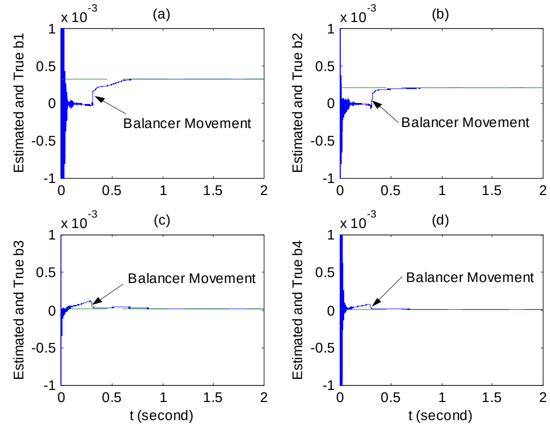

FIGURE 2. ESTIMATION RESULTS. THE TRUE VALUES ARE REPRESENTED BY DASH LINES.

After the system imbalance is estimated, the optimal position of the balancer needs to be calculated and the balancer needs to be moved to that position to offset the imbalance. This balancing strategy will be presented in the next section.

4. CONTROL STRATEGY FOR ACTIVE BALANCING

4.1 Supervisory Adaptive Control Strategy

The actuator used in this study is a mass redistribution device. The best way to minimize the imbalance-induced synchronous vibration is to let the forcing term in Eq. (1) be zero. Confined by the specific mass-redistribution actuator, the conventional control schemes, such as the methods used by Firoozian and Stanway(1988), Stanway and Burrows(1981), and Ulsoy(1984), cannot be used in the active balancing scheme. The vibration suppression ability of the active balancing scheme lies in the elimination of the root cause of the vibration, i.e., system imbalance. The performance of the scheme mainly depends on the accuracy of the estimation of the system imbalance. The explicit relationship between the estimation error (the residue system imbalance after active balancing) and the system residue vibration can be obtained by the system transfer function. The advantage of active balancing over external force control is that the balancer can offer a very large counter imbalance force that is required at high speed, while the external force control scheme cannot.

There are two vibration modes for a rigid rotor system. To control the translational mode, we need

$$w_z = -\frac{m_u u_z}{m_b} \text{ and } w_x = -\frac{m_u u_x}{m_b} , \tag{7}$$

to offset the imbalance-induced force; To control the conical inclination mode, we need

$$w_z = -\frac{m_u u_z u_y}{m_b w_y} \text{ and } w_x = -\frac{m_u u_x u_y}{m_b w_y}, \quad (8)$$

to offset the imbalance-induced moment.

It is clear that if the balancer is not at the same transverse plane as the imbalance, i.e., $w_y \neq u_y$, these two modes cannot be controlled simultaneously by only one balancer. A straightforward way to balance under this situation is to control the currently dominant mode only.

4.2 Simulation Study

Figure 3 shows the simulation results using different control strategies. In this simulation, the imbalance (0.5kg) is located at (0.08, -0.2, 0.05) and the balancer (0.5kg) is located at (w_x, 0.2, w_z), where w_x and w_z are control inputs. In Fig. 3, only w_x is shown. The output is the vibration of a point located at (0, 0.1, 0) in the body fixed coordinate.

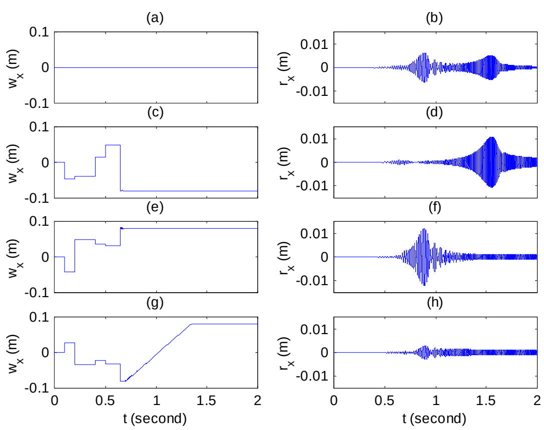

FIGURE 3. DIFFERENT CONTROL STRATEGIES.

Fig. 3(b) shows the uncontrolled vibration. The two vibration modes of the shaft can be clearly seen. Fig. 3(c) shows the simulation result of an active balancing scheme, which integrates the recursive least squares estimation of imbalance and the control law of Eq. (7). The control strategy is that the balancer is varied randomly in the first 0.65 seconds to make the estimation of imbalance converge, then Eq. (7) is used as the control law based on the recursive least squares estimation results. The vibration of the first vibration mode is suppressed by this strategy. However, the vibration of the second vibration mode becomes worse, which is shown in Fig. 3(d). Similarly, the control strategy used in Fig. 3(e) only controls the second vibration mode. The vibration of the first vibration mode becomes worse, as shown in Fig. 3(f).

The control strategy adopted by Fig. 3(g) tries to suppress the vibration of the two vibration modes by only using one balancer. The strategy is to

1. vary the balancer randomly to get good estimation of the position and magnitude of the system imbalance;
2. put the balancer at the opposite side of the estimated imbalance to control the first vibration mode; and
3. change the control strategy to control the second vibration mode starting at 0.7 seconds and ending at 1.35 seconds. During this transition period, a simple linear interpolation function is used.
4. control the second vibration mode only after 1.35 seconds.

Among these steps, the first and third step can be viewed as a supervisory effort. The first one tries to get good estimation. The third step tries to get good control. Comparing the controlled output of these two control strategies, we can see that the supervisory control method (the control strategy used in Fig. 3(g)) can reduce the imbalance-induced vibration efficiently.

This control strategy is easy to implement. The positions of dominant modes can be calculated using the estimated dynamic parameters of the system (*m, c, k*) and the acceleration rate, or can be determined by trial runs. Then the starting switch time (0.7 seconds in the above simulation) can be obtained from the dominant modes positions. This method can be extended to the vibration control of flexible rotor (more than two vibration modes in the system). Since this strategy only tries to offset the equivalent imbalance in current dominant vibration mode, the "spillover" which is a common problem in external force control of flexible structure (Balas, 1978) does not appear.

5. CONCLUSION

An active real-time balancing scheme for a rigid rotor system during transient time of acceleration is proposed in this paper. The speed-varying transient dynamic model of the rigid rotor has been built. With the small motion assumptions, the system is a time-variant linear system. An ordinary recursive least squares estimation method is applied on the transient model to estimate the imbalance in the system. This method uses the transient vibration signal and the constant rotating speed condition is not required. Based on this rigid rotor model and considering the mass redistribution actuator, a simple supervisory adaptive control strategy is proposed. The simulation that integrates the estimation and the control shows that this whole active balancing scheme can reduce the imbalance-induced vibration efficiently. The experimental validation of this active balancing scheme is being carried on. The results will be reported in the future.

One thing that needs to be pointed out is that the starting switch time (0.7 seconds in the simulation), the ending switch time (1.35 seconds in the simulation), and the interpolation function of the switching (a linear function in the simulation) in this supervisory adaptive control strategy are determined qualitatively. To determine these parameters quantitatively, an optimization procedure for these parameters needs to be applied. This work will also be presented in the future.

APPENDIX

We assume the spinning motion about the Y-axis is known and the translational motion in Y direction is zero. By Newton's law,

$$\begin{cases} M_x = I_{xx}\omega_x + (I_{zz} - I_{yy})\omega_y\omega_z \\ M_z = I_{zz}\omega_z + (I_{yy} - I_{xx})\omega_x\omega_y \\ F_x = m\ddot{R}_X \\ F_z = m\ddot{R}_Z \end{cases} . \quad \text{(A-1)}$$

There are three kinds of forces applied to the rigid shaft: elastic force, damping force, and inertia force caused by the imbalance.

(1). Elastic force and moment $\mathbf{F_k} = -2k[R_X, R_Z]^T$ and $\mathbf{M_k} = [-\frac{1}{2}kL^2\theta, -\frac{1}{2}kL^2\psi]^T$.

(2). Damping force and moment $\mathbf{F_c} = -2c[\dot{R}_X, \dot{R}_Z]^T$ and $\mathbf{M_c} = [-\frac{1}{2}cL^2\dot{\theta}, -\frac{1}{2}cL^2\dot{\psi}]^T$.

(3). Inertia force and moment induced by imbalance

$$\mathbf{F_i} = -m_u d\begin{bmatrix} \ddot{\phi}\sin(\alpha-\phi) - \dot{\phi}^2\cos(\alpha-\phi) \\ -\dot{\phi}^2\sin(\alpha-\phi) - \ddot{\phi}\cos(\alpha-\phi) \end{bmatrix}, \text{ and}$$

$$\mathbf{M_i} \approx \begin{bmatrix} m_u d(\dot{\phi}^2\sin(\alpha-\phi) + \ddot{\phi}\cos(\alpha-\phi))u_y \\ m_u d(-\dot{\phi}^2\cos(\alpha-\phi) + \ddot{\phi}\sin(\alpha-\phi))u_y \end{bmatrix}.$$

The imbalance mass m_u is located at $[d\cos\alpha \;\; u_y \;\; d\sin\alpha]^T$ in the body-fixed coordinate. Substituting these terms into Eq. (A-1) and noting $u_x = d\cos\alpha$, $u_z = d\sin\alpha$, we can get the governing equation of rigid rotor used in section 2.

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REFERENCE

Balas, M. J., 1978, "Feedback Control of Flexible Systems", *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 4, Aug. pp673-679.

Dyer, S. W., Hackett, B. K., and Kerlin, J., 1998, "*Electromagnetically Actuated Rotating Imbalance Compensator*", U.S. Patent No. 5,757,662, May 26.

Firoozian, R. and Stanway, R., 1988, "Modeling and Control of Turbomachinery Vibrations", *ASME, Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 110, Oct. pp521-527.

Genta, G., 1993, "*Vibration of Structures and Machines: Practical Aspects*", Springer-Verlag.

Gosiewski, Z., 1985, "Automatic Balancing of Flexible Rotors, Part 1: Theoretical Background", *Journal of Sound and Vibration*, 100(4), pp551-567.

Gosiewski, Z., 1987, "Automatic Balancing of Flexible Rotors, Part 2: Synthesis of System", *Journal of Sound and Vibration*, 114(1), pp103-119.

Knospe, C. R., Hope, R. W., Tamer, S.M., and Fedigan, S. J., 1996, "Robustness of Adaptive Imbalance Control of Rotors with Magnetic Bearings", *Journal of Vibration and Control*, 2:33-52.

Ljung, L. and Soderstrom, T., 1983, "*Theory and Practice of Recursive Identification*", The MIT Press.

Lum, K. Y., Coppola, V. T. and Bernstein, D. S., 1996, "Adaptive Autocentering Control for an Active Magnetic Bearing Supporting a Rotor with Unknown Mass Imbalance", *IEEE Transactions on Control Systems Technology*, Vol. 4(5), Sept. pp587-597.

Stanway, R. and Burrows, C. R., 1981, "Active Vibration Control of a Flexible Rotor on Flexibly-Mounted Journal Bearings", *ASME, Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 103, Dec. pp383-388

Ulsoy, A. G., 1984, "Vibration Control in Rotating or Translating Elastic Systems", *ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol 106, Mar. pp6-14.

Van De Vegte, J. and Lake, R. T., 1978, "Balancing of Rotating Systems During Operation", *Journal of Sound and Vibration*, 57(2), pp225-235.

Van De Vegte, J., 1981, "Balancing of Flexible Rotors During Operation", *Journal of Mechanical Engineering Science*, Vol. 23(5), pp257-261.

Wowk, V., 1995, "*Machinery Vibration: Balancing*", McGraw-Hill, Inc.